Classification of Finite Type Laurent Phenomenon Algebras

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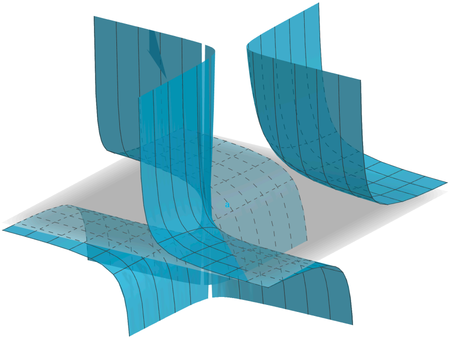


What is a Laurent phenomenon algebra?

- Let K be rational function field in n independent variables over $\mathbb{C}$, $x_1, \ldots, x_n$ trsc. basis for $K/\mathbb{C}$, $f_1, \ldots, f_n$ irred. polys in $x_1, \ldots, x_n$ such that f_i does not depend on x_i and is not divisible by any x_j, $j \neq i$.
- **Seeds:**

$$S = (\mathbf{x}, \mathbf{F}) = \Big(\{x_1, \ldots, x_n\}, \{f_1, \ldots, f_n\}\Big) \qquad (1)$$

  - Cluster ($\mathbf{x}$)
  - Exchange polynomials ($\mathbf{F}$)
  - Cluster variables ($x_1, \ldots, x_n$)
- **Mutations**: family of involutive maps μ_i defined on seeds by

$$(\{x_1, \ldots, x_i, \ldots, x_n\}, \{f_1, \ldots, f_i, \ldots, f_n\})$$
$$\downarrow \mu_i$$
$$\big(\{x_1, \ldots, x_i', \ldots, x_n\}, \{f_1', \ldots, f_i, \ldots, f_n'\}\big)$$

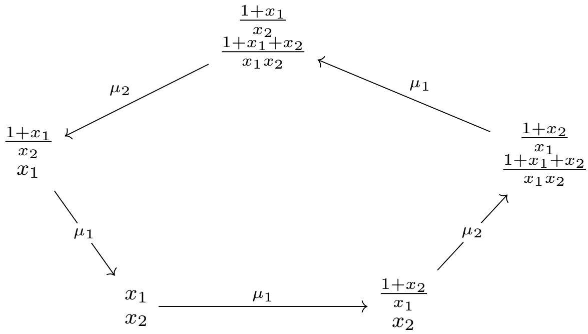
  where $x_i x_i' := f_i$ (up to uniquely determined Laurent monomial multiplier), and f_j' are uniquely determined by seed (up to a unit multiplier).

Definition (Lam-Pylyavskyy, 2012)

The *Laurent phenomenon algebra* (LP Algebra) $\mathcal{A}(S)$ associated to S is the $\mathbb{C}$-algebra generated by all cluster variables obtainable through mutation.

Rank two example

- S = seed with exchange polynomials $f^1 = 1 + x_2$, $f^2 = 1 + x_1$.



- We have *finitely many seeds!* By definition

$$\mathcal{A}(S) = \mathbb{C}\left[x_1, x_2, \frac{1+x_1}{x_2}, \frac{1+x_2}{x_1}, \frac{1+x_1+x_2}{x_1 x_2}\right].$$

- $\mathcal{A}(S) = \mathbb{C}[x,y,z]/(xyz - x - y - 1)$ is coordinate ring of a hypersurface X in $\mathbb{A}^3$.
- **Laurent phenomenon**: *Every cluster variable is a Laurent polynomial in the initial cluster.*

Definition (Finite type)

If the set of seeds obtainable by mutation of seed S is finite then we say S is of *finite (mutation) type*.

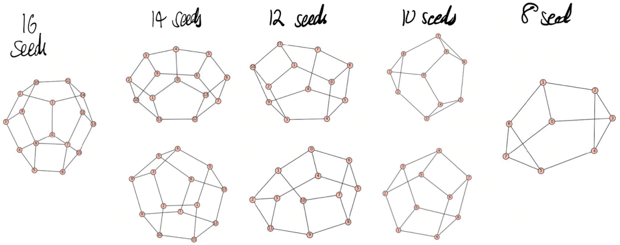
Definition (Linear seed)

If all variables in the initial exchange polynomials in seed S appear with degree at most 1, then we say S is *linear*.

Theorem

All linear LP algebras are finite type.

- Degenerating coefficients of a linear seed corresponds to degenerating the face structure of the exchange graph.



Thanks for listening!